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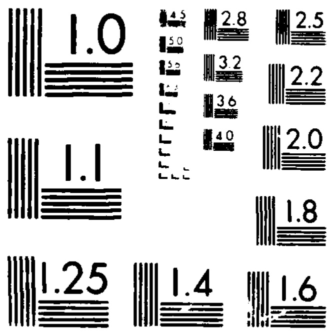
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This Final Report summarizes the research conducted at the University of Illinois at Chicago by the author and several of his graduate students. The research was focused on the establishment of a rational constitutive model for concrete based on the actual mesostructural kinetics. The results clearly demonstrate that a constitutive model of the proposed type is not only feasible but also that the attendant computations are moderate in volume.
The proposed model is illustrated on several simple examples in the referenced literature.

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FINAL REPORT

MICROMECHANICS OF CONCRETE

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January 1988

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I. INTRODUCTION

With a present annual consumption estimated at 4.5 billion tons (Kumar Mehta, 1986), concrete undoubtedly ranks at the very top of the list of most popular materials. The availability of the constituent materials, economy of production (often at the site), simplicity of the manufacturing procedures, formability and a bevy of other desirable properties present compelling reasons for the continuing popularity of concrete.

On the debit side, a relatively modest tensile strength of concrete is a limiting factor with regard to utility and a cause of concern in design and utilization. Even before its ultimate demise, the nonlinear response of a concrete structure (i.e., gradual deterioration of its strength) is traceable to the growth of a large number of microcracks distributed rather evenly over a large part of the volume (damage). After some early and futile attempts to modify the venerable theory of plasticity for the analysis of concrete structures, it became obvious that a rational constitutive model must be based on the physical reality and account for microcracking as the dominant aspect of the deformation process.

The title continuum damage mechanics (CDM) is often used to label a loose agglomeration of analytical models formulated with an express purpose of dealing with brittle response of solids. The continuing reliance of these models on the macro observations resulted in a host of different and often contradicting theories. With a generous helping of 'material' parameters, these phenomenological models are shown to a fit a particular set of experimental data with an often suspicious accuracy. The ensuing controversy, related to the selection of the 'best' or even most appropriate model for a particular problem, was typically approached using clever little artifices emphasizing unrealistic assumptions regarding crack distribution.

This state of affairs (summarized briefly in Krajcinovic 1984), typical of infancy of most branches of continuum mechanics, has nevertheless ran its course. The recently emerging micromechanical models (M. Kachanov 1982, 1987, Horii and Nemat-Nasser 1983, 1986, Krajcinovic and Fanella 1986, Krajcinovic and Sumarac 1987, t.a., Sumarac and Krajcinovic 1987, etc.) suggest in no uncertain manner that most of the existing phenomenological models stand on a very shaky ground. This, of course, just proves that every attempt to model an irreversible process on the basis of macro observations presents a risky venture having a modest chance of successful outcome.

The objective of this research program was to examine the essential structure of the micromechanical (or, more accurately, mesomechanical) models outlining the assumptions needed to ensure their tractability and, consequently, their practical utility. Even more importantly, the investigation was focused on the mechanical response of the concrete and use of these micromechanical theories to establish reliable analytical tools for the prediction of the response of concrete in a wide set of circumstances.

II. ANALYTICAL MODEL

II.1 <u>The Question of Scale and the Damage Variable</u>

The early efforts in developing the CDM theories for dealing with brittle processes were almost exclusively phenomenological. One of the basic tenets which both divided and unified these efforts was that it was deemed necessary to divine the mathematical representation of a somewhat nebulous 'damage (internal) variable' up front. None clung to this 'principle' more tenaciously than those enamored with constructing little clever stratagems demonstrating perceived advantages of a particular representation. The ensuing confusion in having to decide upon the 'best', or at the very least the most suitable, of these theories could not be underestimated (see, Krajcinovic 1984).

Not surprisingly the answer to this thorny question required an inquire into the physics of the phenomenon and the structure of the solid. In order to define the damage, it turns out to be necessary to consider two scales: macro-scale on which the solid is regarded as continuum and the damage is represented by a continuous function of coordinates and the meso-scale on which the solid is inhomogeneous and some of the fields discontinuous. The introduction of two scales implies existence of the unit cell, or representative volume, (Hill 1967) which, loosely speaking, presents the smallest volume of material which in a given sense responds as a macro-continuum. Mathematically, a unit cell maps on the material point of the continuum. The configurational space (recorded history H) attached to each point of the continuum must, therefore, contain necessary information defining [9]*:

- number, type, position, size and orientation of defects,
- number and orientation of cleavage and slip planes, and
- morphology of energy barriers (such as aggregate facets) within the unit cell.

In contradistinction to ductile deformation, during which the material 'slips through' the crystalline lattice, the damage can be defined as a process during which the material remains 'in place' while the lattice degrades through the loss of bonds. Since the elastic compliance of a solid locally depends on the type, distribution and density of primary bonds, it is not surprising that it changes only in the course of brittle processes. In fact, in a complete accord with the original Kachanov's (1958) model the extent of dam-

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*The number in brackets refers to the list of papers summarized in this Report (see Chapter VI).

age can be measured by the change in the elastic macro-compliance of the solid (Chaboche 1979, Lemaitre and Chaboche 1985). This characterization is also suggested by Hart, et al. (1975) who argued that the change in compliance being a readily identifiable and measurable property of the specimen is a natural choice for the internal variable.

On the meso-scale, the ruptured bonds cluster into cracks along the aggregate facets. Assuming that each of N planar cracks within a unit cell mapping on a point $\underset{\sim}{x}$ can be regarded as elliptical the configurational space (or history H) defining the 'damage' in the point contains 8N variables: three scalars defining position, three Euler angles defining orientation and two scalars defining size of each crack.

A much less ambitious description of damage involves only 4N variables assuming cracks to be circular and disregarding their exact position within the unit cell. The 'damage' is then described by N doublets $\underset{\sim}{\omega}\,(\underset{\sim}{x};\, A,\, \underset{\sim}{n})$, where A is the relative void area in the plane with normal $\underset{\sim}{n}$ through the point $\underset{\sim}{x}$ (Krajcinovic 1985, Ilankamban and Krajcinovic t.a.).

An even simpler characterization is possible when the crack distribution and orientations are perfectly random rendering the solid isotropic. In that case, a single scalar 'crack density parameter' (Budiansky and O'Connell 1976, Lemaitre and Chaboche 1985) suffices. Every successive approximation enhances the tractability at the expense of the rigor and possibly accuracy.

An important aspect of the definition of 'damage parameter' is related to the unilateral constraint which a crack places on the displacement field. Consequently, all cracks must be divided into open and closed cracks (Horii and Nemat-Nasser 1983, Krajcinovic 1985, etc.). In the first case, the displacement field is discontinuous while the closed cracks have no effect on the displacement and, therefore, compliance as well.

II.2 Thermodynamic Considerations

Consider a perfectly elastic material in a completely inert environment. Restricting discussions to perfectly brittle deformation processes, the current state is defined locally by the temperature T, elastic strain tensor and a history parameter H.

According to the first law of thermodynamics, the sum of rates of changes in the mechanical work dW and heat energy dQ must be balanced by the sum of rates of changes of the internally stored energy du and the energy needed for the creation of new, internal surfaces (rupture of bonds) dD, i.e.,

$$dW + dQ = du + dD \tag{1}$$

where all terms in (1) are taken per unit of volume.

Introducing the dual (Gibbs) potential density function ψ, the entropy production (Clausius-Duhem) inequality can be, for $dT = \text{grad } T = 0$, written as

$$d^{I}\psi - dD \geq 0 \tag{2}$$

where the superscript (I) denotes irreversible change.

In accord with the adopted definition of the damage, the energy dissipated on the creation of new internal surfaces is proportional to the increase in the crack surfaces. Thus, denoting by $\delta\ell(L)$ advance of the crack front L in the direction of its normal, the energy dissipated on the creation of new surfaces can be written in form (Rice 1975)

$$dD = \int_{L} 2\gamma \; \delta\ell \; dL \tag{3}$$

where γ is the surface energy.

Finally, since the energy release rate G (Rice 1978) is defined as

$$G = -(\partial\psi/\partial\ell) \tag{4}$$

the inequality (2) can be rewritten in a more familiar form (Rice 1975) as

$$\int_L (G-2\gamma)\ \delta\ell\ dL \geq 0 \tag{5}$$

where $G = J$ is the energy release rate density. The inequality (5) is typically assumed to be satisfied pointwise on L, leading to the more common form of the stability criterion

$$(G-2\gamma)\delta\ell > 0 \tag{6}$$

valid both for rapid cracking and healing. In the language of thermodynamics, $\delta\ell$ is the flux, G the affinity (driving force) and 2γ the resistive force. Consideration of a more general crack growth pattern will result in the increase of the number of fluxes and their conjugate forces (Chudnovsky 1987).

II.3 Process Model

The essential form of a rational analytical model for the analysis of the brittle response of a solid is dictated by the conflicting requirements of computational facility and simplicity on one and the rigor in modelling the underlying physics of the phenomenon on the other hand. The nonlinearity in the macro response of the solid is obviously just the reflection of the irreversible rearrangements of the mesostructural fabric of the solid. Consequently, a realistic theory must include a rational model of the dominant mechanism of these irreversible changes as a basic building block. Since the mesostructural fabric is almost never deterministic in nature, such a model will rest on appropriate stochastic distributions of the state and internal variables of the problem.

However, for the reasons of computational efficiency our models exhibit strong preference for the deterministic continua and gradually varying continuous functional representations of all variables.

Consequently, a model satisfying both sets of requirements will consider both scales. The mesomechanical models of the dominant energy dissipating mechanisms are (via an appropriate homogenization algorithm) embedded into a deterministic relation mapping macro stresses on macro strains in an effective continuum. This effective continuum approximates in a physically justifiable sense the actual solid with locally discontinuous fields.

A process model includes both the descriptions of the state (compliance tensor) and the manner in which the state changes (kinetic equations or the 'damage law'). Even though related these two tasks are sufficiently different and will be discussed independently.

II.4 Compliance Tensor

Owing to a large number of stochastic parameters (8N) needed to define the 'damage' within the representative volume mapping on a material point $\underset{\sim}{x}$ of the effective macro-continuum, it is apparent that the establishment of an analytical model claiming practical utility requires introduction of some simplifying assumptions. Some of these assumptions are neither very restrictive nor controversial. For instance, on the basis of available computations, it is commonly accepted that the ellipticity of the microcracks does not have a strong effect on the compliance. Consequently, for all practical purposes every microcrack can be assumed as being circular in shape without a serious loss of accuracy.

A more restrictive assumption is associated with the application of the effective field theories (Mura 1982, Kunin 1983, etc.) according to which the external field of each crack weakly depends on the exact position of other cracks. The overall macro compliance of the solid can be then written in form

$$\underset{\sim}{S}(\underset{\sim}{x},H) = \underset{\sim}{S}^{o}(\underset{\sim}{x}) + \sum_{k} \underset{\sim}{S}^{*}_{(k)}(\underset{\sim}{x},H) \tag{7}$$

where $\underset{\sim}{S}^{o}(\underset{\sim}{x})$ is the compliance of the virgin material (which might reflect the anisotropy other than that introduced by the damage), while H is the recorded history (reduced here to 3N angles and N radii) defining the 'damage'. The second term in the above expression defines the compliance attributed to N open cracks within the representative volume. Within the framework of the effective field theories [1] each crack can be considered as a single crack embedded into a homogeneous (effective) continuum which in some 'self-consistent' and smoothed sense reflects the presence of other (N-1) cracks. The domain of validity of this class of methods is restricted to the cases characterized by modest levels of the direct interaction between the adjacent cracks, i.e., to low and moderate crack concentrations.

The effective field theories, in conjunction with the Eshelby inclusion method, allow for a relatively straightforward determination of the overall compliance of the microcrack weakened solid whenever the solutions for the corresponding eigenstrains are available. Specifically, the work performed within this project was focused on the two levels of approximation:

- Taylor model (TM) which completely ignores the crack interaction assuming that each crack is embedded into the virgin (undamaged) solid, and

- the self-consistent model (SCM) which assumes that the influence of the adjacent cracks is sufficiently well specified by the volume density and orientation of the adjacent cracks and that the external stress field of each crack is equal to the macro-stress.

In the case of the TM, a closed-form, analytical solution for the eigenstrains (i.e., crack opening displacement) is available whenever the solid is originally isotropic. Consequently, the TM is very appealing in application even though its accuracy might be questionable in case of larger crack concentrations. It is obvious, though, that the Taylor model provides the lower bound on the strains since it overestimates the rigidity of the solid ignoring the adjacent cracks. The SCM model while obviously more accurate leads to much more difficult computational algorithms. First of all, the analytical expressions for the crack opening displacements are available only for the isotropic and transversely isotropic solids. Secondly, even in these cases the crack opening displacements (eigenstrains, see Hoenig 1978,1979, Sih, et al. 1965, etc.) depend both explicitly and implicitly on the compliance of the effective solid. Hence, a purely analytical solution is not possible and even the numerical solution requires application of an iterative algorithm similar to that familiar from the slip theory models.

The principal advantage of the TM and SCM is that the total (macro) strain in a material point (on which the corresponding representative volume is mapped) can be written in the form of a sum

$$\underset{\sim}{\varepsilon} = \underset{\sim}{\varepsilon}^{o} + \sum_{k=1}^{N} \underset{\sim}{\varepsilon}^{*}_{(k)} \tag{8}$$

where $\underset{\sim}{\varepsilon}^{*}_{(n)}(\underset{\sim}{x})$ is the eigenstrain 'in' the n-th crack, while the sum extends over all N open cracks in the representative volume. Also, $\underset{\sim}{\varepsilon}^{o}(\underset{\sim}{x})$ is the elastic strain related to the average (macro) stress through the familiar equation

$$\underset{\sim}{\varepsilon}^{o} = \underset{\sim}{S}^{o} : \underset{\sim}{\sigma} \tag{9}$$

The eigenstrain 'in' the n-th crack, averaged over the volume V of the unit cell, may be written (Mura 1982, Kunin 1983, M. Kachanov 1982, Horii and Nemat-Nasser 1983, [4,5,9], etc.) as

$$\varepsilon^{*}_{ij(n)} = \frac{1}{2V} \int_{A_{(n)}} (b_i n_j + b_j n_i)\, dA \tag{10}$$

where $A_{(n)}$ is the area of the crack surface, $\underset{\sim}{n}$ the normal to the crack surface and $\underset{\sim}{b}$ the displacement discontinuity vector (crack opening displacement). The eigenstrains can, at least in principle, be always written in terms of the average stresses (Hoenig 1978, Mura 1982)

$$\underset{\sim}{\varepsilon}^{*}_{(n)} = \underset{\sim}{S}^{*}_{(n)} : \underset{\sim}{\sigma} \tag{11}$$

Consequently, the overall compliance $\underset{\sim}{S}(\underset{\sim}{x},H)$ can indeed be readily written as a juxtaposition of compliances attributable to individual cracks (12) and the original, elastic compliance (10) of the matrix, in a form suggested by (7).

The major problem consists of the determination of the displacement discontinuity vector $\underset{\sim}{b}$ which is needed in the computation of the eigenstrains from (11). Whenever a closed-form analytical (exact or approximate) solution for $\underset{\sim}{b}$ exists the proposed model can readily be applied. However, at the present time the solutions for the crack opening displacement exist only for cracks embedded in isotropic or transversely isotropic solids (Sih, et al. 1965, Hoenig 1979). In all other cases the crack opening displacements can be determined only after protracted and non-trivial arduous computational effort (Mura 1982). In some cases (Horii and Nemat-Nasser 1986, [8]) it is possible to find surprisingly accurate approximate expressions for the crack opening displacements $\underset{\sim}{b}$ for rather complex crack geometries (kinked cracks).

The crack opening displacements in the case of isotropic solids (TM) can be written [2] for a penny-shaped crack of radius a

$$b_i' = (a^2-r^2)^{1/2} B_{ij}'\sigma_{3j}' \tag{12}$$

where $r \leq a$ is the radial coordinate, while the primes indicate the reference to the local coordinate system embedded in the crack such that n_3' coincides with the normal to the crack surface. The tensor $\underset{\sim}{B}$ can then be written in a simple form

$$B_{ij}' = 8\,\frac{1-\nu^2}{\pi E}\left[\delta_{i3}\delta_{j3} + \frac{2}{2-\nu}\,(\delta_{i2}\delta_{j2} + \delta_{i3}\delta_{j3})\right] \tag{13}$$

where $\underset{\sim}{\delta}$ are the Kronecker-delta symbols, ν Poisson's ratio and E the elastic modulus of the isotropic, elastic solid.

The global (unprimed) an local (primed) coordinate systems are related by means of the conventional rotation matix

$$n_i' = g_{ij}n_j \tag{14}$$

The components of $\underset{\sim}{g}$ are the sines and cosines of the Euler angles subtended by the axes of the global and local coordinate systems. Substituting (13) and (14) into (8) the compliance attributable to the crack opening can be written in a very simple form as

$$S^*_{ijmn} = \frac{8(1-\nu^2)}{3VE(2-\nu)} \sum a^3(\delta_{im}n_jn_n + \delta_{in}n_jn_m + \delta_{jm}n_nn_i + \delta_{jn}n_in_m - 2\nu n_in_jn_mn_n) \tag{15}$$

In practical applications since the number of cracks N is typically very large, the sum in (15) is recast into a triple integral using the probability density functions for the crack orientations and sizes. These functions can be readily established from the elementary considerations of the aggregate geometry [2,4,5,8,9].

The determination of the compliance attributable to open cracks is a great deal more complicated in the case of the SCM. Since simple enough solutions for the crack opening displacements exist only for the two-dimensional case of transverse isotropy (Sih, et al. 1965) the applications are restricted only to the simple stress fields. Even in the case of the transverse isotropy the crack opening displacements are both implicit and explicit functions of the total compliance of the solid. The implicit dependence involves the roots of the fourth order characteristic equation (Sih, et al. 1965, Horii and Nemat-Nasser 1983, [5,9]). Using a special transformation the characteristic equation was reduced in [5,9] to a biquadratic equation admitting analytical solutions for the roots. Nevertheless, the determination of the vector b involves guessing the anisotropy ratio $m = S_{22}/S_{11}$ between the axial and lateral compliance. Once b is determined on the basis of the initial guess for m, the compliances and the ratio m can be computed in a closed form for every value of the externally applied load. If the predicted and computed values for m are sufficiently close the computation can proceed to the subsequent load increment.

II.5 Kinetics of the Crack Growth

The final step in the formulation of the constitutive law consists of the determination of a functional relationship (damage law) between the increase of the microcrack radius and the increment of the crack driving force (energy release rate G). Typically, a damage law is deduced on the basis of hypothetical arguments based almost exclusively on macro observations. However, as shown by Krajcinovic and Fanella (1986), a constitutive relationship can actually be derived considering stochastic distributions of weak planes and energy barriers on the mesoscale.

In determination of the crack growth pattern of a meso-crack, the stochastic distribution of surface energies (geometry of cleavage planes and grain boundaries) plays a major role. In principle, as the stress levels are gradually raised, a crack can grow (penetrate successive energy barriers) either in a steady (smooth) mode consisting of an infinite number of continuously changing values (σ,ℓ) satisfying (6) or in a jerky manner emphasizing long periods of languor $(\delta\ell = 0)$ interspersed by short, almost instantaneous bursts of rapid changes in the crack size ℓ. The latter case appears to be more common in the case of brittle and semi-brittle solids in predominantly tensile regimes. The acoustic signature of the energy ΔD released during the rapid increase in the crack size is readily recorded in tests (Holcomb and Costin 1986).

The crack stability criterion is derivable directly from the entropy production inequality (6). The crack growth is stable if

$$\frac{\partial G}{\partial \ell}\,\delta\ell < 0 \qquad (16)$$

and unstable when

$$\frac{\partial G}{\partial \ell}\,\delta\ell > 0 \qquad (17)$$

In the first of the two cases a crack will grow in a steady mode as the stress increases. The second case is characteristic of jerky changes in the crack size.

More specific arguments leading to the formulation of kinetic equations for concrete will be discussed in the sequel.

III. APPLICATION TO CONCRETE

III.1 Microstructure of Concrete

Structurally, concrete is a particulate composite of stone and sand aggregate embedded in the cement paste matrix serving as a binding agency. Disregarding the finer points of its chemical compositions and chemical reactions during the hardening process for the present purposes it suffices to state that the mechanical response of concrete directly reflects:

(i) its composition, i.e., toughnesses of the constituent phases and the dispersion, volume fraction, size and shape (sieve gradation) of the coarse aggregate as well as its surface texture, and

(ii) the size and distribution of the initial cracks (and porosity of the matrix) attributable to the hydration heat and free water migration (popularly classified as shrinkage, bleeding, creep, aging, etc.).

The mesostructural morphology and fabric of concrete is to a large extent responsible for the pattern of the microcrack (or 'damage') evolution. The so-called transition zone, consisting of relatively large crystals of ettringite and calcium hydroxide (Kumar Mehta 1986), is the weak link of the composite. The relatively high porosity and plate-like form of the hydroxide crystals are reflected through inferior levels of surface energy. Additionally, the stress concentrations resulting from the disparate heat conduction and elastic parameters of the aggregate and cement paste further promote the transition zone as a preferential site for the nucleation and path of propagation of microcracks.

The existence of interfacial cracks before the application of mechanical loads would classify concrete as a 'cleavage 1' solid (Ashby 1979). It seems safe to assume that most of the cracks are already present in concrete and that the difference between the imparted and stored energies during the exploitation is dissipated primarily on the growth of the already existing interfacial cracks.

The scenario depicting the crack growth patterns depends on the morphology of the mesoscale and the stress field. As already mentioned, a microcrack will start increasing in size if and only if the instability criterion (7) is satisfied. The energy release rate (driving force) G is a function of the macro-stress field in the region surrounding the crack tip. Thus, the determination of $G = J$ intrinsically involves averaging (associated with the integration) and is, therefore, less susceptible to local fluctuations of stresses. In contrast, the surface energy γ is a random variable characterized by significant variations from some (often meaningless, see papers in Wittmann 1983) average values associated to the macro specimen.

The distribution of the surface energy reflects the dispersion of the coarse aggregates on the mesoscale. Specifically, the experimental measurements indicate (Mindess and Young 1981, Zaitsev 1982 and 1983, etc.) that for a typical concrete mixture

$$\gamma_a \gg \gamma_c \simeq 2\gamma_{tz} \tag{18}$$

where the subscripts (a), (c) and (tz) refer to the aggregate, cement paste and the transition zone, respectively.

Consequently, an interfacial crack for which $G > 2\gamma$ will become unstable and start propagating in a runaway fashion along the interface (through the transition zone) until it reaches the edge of the facet. At this point the crack must either kink onto the sequent interface (following the aggregate contour) or commence propagation through the cement paste having superior surface energy (fracture toughness). Thus, a crack is trapped at the aggregate facet edge either by the decreasing driving force (associate with kinking) or by the increasing resistance. In either case additional energy must be imparted externally for the crack to continue its growth.

According to this scenario, each crack grows in a spasmodic fashion reminiscent of the jerky motion of dislocation across a slip plane of inhomogeneous slip resistance. The magnitude of the driving force G depends on the stress field, i.e., on the crack radius and it orientation with respect to the principal stress axes. Since both the crack size and orientation are random variables the cracks will become unstable sequentially as the load is incremented. This conclusion was confirmed experimentally through acoustic emission tests (see the paper by Diederich, et al., in Wittmann 1983 and Holcomb and Costin 1986). Hence, the energy is indeed dissipated not continuously but in a sequence of successive bursts associated with sudden jumps in the size of individual microcracks.

The basic idea and the simple elegance of the proposed derivation of kinetic equations can best be illustrated on the case of uniaxial tension [2,9]. Neglecting, for simplicity, the Mode II cracking the stability criterion (6) for the k-th crack within the representative volume can be written using the stress intensity factors as

$$f_{(k)} = K_{I(k)} - K_{IC} = 0 \tag{19}$$

The critical stress intensity factor K_{IC} is a random function which follows the ratios for the surface energies (18). Thus the elastic region $f < 0$ is a polyhedron defined by hypersurfaces (19) in the stresss space. Furthermore, in the observed case, for a penny-shaped crack with a normal $\underset{\sim}{n}$ subtending angle θ with the tensile axis the expression (19) reads

$$f_{(k)} = 2q \cos^2\theta\sqrt{\frac{a}{\pi}} - K_{IC} = 0 \tag{20}$$

where q is the externally applied traction.

Consequently, an original interfacial crack of radius a_o for which the inequality

$$q \cos^2\theta \sqrt{a_o} \geq \frac{\sqrt{\pi}}{2} K_{IC} \tag{21}$$

is satisfied will become unstable and commence unstable propagation through the transition zone (along the aggregate facet) until it gets arrested at the facet edge by the cement paste having superior toughness (18).

Thus, at each value of the externally applied tensile tractions q, it is possible to distinguish the cracks which will retain their original size ($f < 0$) from those which will occupy the entire surface of the facet ($f > 0$). The hypersurface $f = 0$ is depicted in the space (q,a,θ) in Fig. 1. The cracks which already changed their length are defined by doublets (a,θ) belonging to the upper right corner of the cube defining all possible crack geometries (D_M is the diameter of the largest aggregate facet). Once the crack radii are known for all orientations and loads, the compliance is computated from (15) and (7) or similar relations in the case of SCM [5,9].

Since the number of microcracks is large, and since their orientations and sizes vary almost continuously over wide ranges, the large number of events on the mesoscale following each other in a close sequence translates into a relatively smooth and gradual change of the macro-response (decrease of stiffness) observable in experiments. This lends the credence to the wisdom of the fundamental strategy on which the proposed methodology is based.

III.2 Failure Modes

Succinctly stated the ultimate failure of concrete (Rudnicki 1979, Horii and Nemat Nasser 1986, etc.) can occur either as a result of the runaway propagation of a single, preferentially oriented and located, microcrack or as a result of a localization process during which the driving force is rapidly enhanced through the interaction of closely spaced microcracks. Generally speaking, in the case of homogeneous (or nearly homogeneous) stress fields for which at least one of the principal stresses is tensile, the energy release rate G of the critical crack will exceed the largest energy barrier in its path and commence unstable growth leading ultimately to the macro-failure. In such a case, the crack density during the entire deformation process including the part at the incipient failure is typically low.

In absence of the tensile stresses, characteristic of the compression of the laterally confined concrete specimens subjected to compressive stresses, each crack grows in a stable fashion with the increasing stresses. Consequently, the sudden and rapid onset of instability must be attributed to the nonlinear increase of the driving force G related to the interaction of the approaching microcracks. The eventual macro-failure features a thin band of closely-spaced interacting microcracks which eventually coalesce into a macrocrack of critical size. In sharp contrast to the preceding case the crack density, within these bands just prior to the onset of localization, is typically high and an appropriate analysis requires incorporation of the direct crack interaction into the model.

The failures in the cases of inhomogeneous state of stress will, in general, involve even more complicated crack growth patterns. The discussion of these cases is considered beyond the scope of this Report.

III.3 Uniaxial Tension

The initial effort in the formulation of the proposed model was centered on the uniaxial tension of a prismatic concrete specimen [2]. To test the feasibility of the strategy only the TM was considered initially, assuming a limited, uniform probability density function for the aggregate facet sizes. The final relation between macro stresses and macro strains was derived in closed form. This enabled a straightforward determination of the influence of the sieve grading (difference in size between the largest and smallest aggregate) and initial defect distribution (due to bleeding) on the response.

It is important to note that the formulation of the model did not require a single new (additional) and experimentally identifiable material parameter. In addition to the elastic moduli the implementation of the model requires only the knowledge of the critical stress intensity factors for the cement paste and the transition zone, volume fraction of the coarse aggregate and the distribution of its sizes. Unfortunately, the existing experimental data seldom if ever document the latter data. Thus, the close fit between the experimental data and computations obtained for a specific case of a given volume fraction could not be tested over a wider range of parameters.

The final macro-failure was assumed to occur as soon as the first crack starts propagating through the cement paste. This assumption was supported by a simple analysis, based on the small particle statistics, according to which the mean free path between the adjacent particles was found to be rather large [10]. Consequently, it was determined that a crack propagating through the cement matrix will be rather large at the moment when it encounters an aggregate in its path. While possible, it is deemed unlikely that such a crack can be arrested at that point. It is also important to notice that the crack density at the incipient macro-failure was found to be quite moderate reducing the possible significance of the crack interaction.

The stress-strain curve for a specific magnitude of initial (bleeding) defects ρ and the ratio γ of the smallest to largest aggregate facet is shown in Fig. 2. The apparent Poisson ratio, reflecting relative 'softening' (anisotropy) in the axial direction, is plotted in Fig. 3. The dependence of the total strain on the size of the initial (bleeding) defects is depicted in Fig. 4.

While working on this problem, it became obvious that the proposed model by its very nature presents a natural choice for the consideration of the size effect. The driving force G is proportional to the size of the crack and inversely proportional to the angle subtended by the normal to the crack surface and the tensile axis. The probability that a large crack will be found in a plane perpendicular to the tensile axis is obviously proportional to the volume of the specimen (number of cracks). Consequently, it was only necessary to determine the joint probability function relating the crack sizes and orientations with the number of samples (cracks in the specimen). The ensuing results [6] proved to be in close qualitative and even quantitative agreement with the experimentally observed trends.

The computations of the size effect according to the developed model (triangles) are compared with some available experimental data (squares) and a widely used empirical relation (hexagons) in Fig. 5. In view of the relative simplicity of the proposed model the obtained fit is quite remarkable.

The simplicity of the TM and the availability of simple, analytical relations between the macro-stresses and macro-strains is a very appealing feature in practical computations. Nevertheless, it was strongly felt that a rational estimate of the accuracy of the TM (i.e., estimate of the error associated with the total neglect of the crack interaction) was needed. Thus, it became necessary to use the SCM for the case of the uniaxial tension and determine the accuracy of the simpler TM based theory.

As expected, the SCM based theory becomes much more complex [4,5,9,11] and not amenable to a closed-form, analytical solution. In fact, the derivation had to be limited to the plane stress and plane strain case for which the analytical expressions for the energy release rate G and the crack opening displacements could be derived (see Sih, et al. 1965, and Hoenig 1979). Even though these expressions for a single crack were obtained in a closed form, the implicit and explicit dependence on the overall compliance (crack induced anisotropy) necessitated use of an iterative predictor-corrector scheme to compute the mechanical response. Fortunately, in the considered case of uniaxial tension the convergence was found to be very rapid and the computational effort rather trivial. This, of course, will not be true in the case of non-proportional loading programs during which the status of individual cracks could change from open to closed and back during the deformation process.

Additionally, it was possible [5,11] to determine simple upper and lower bounds on the total strain and relate the proposed model to the well-known Budiansky, O'Connell (1976) theory for the solids weakened by an isotropic distribution of cracks.

The normalized stress-strain curves computed according to TM and SCM are plotted in Fig. 6. Despite rather large errors inherent to the simplifications on which the TM is based it must be kept in mind that, in the case of concrete, the final rupture load seldom, if ever, exceeds the load q_o at the onset of the nonlinear response (first cracking) by a factor of more than 2.5. For $q/q_o = 2.5$ the error of the TM is barely 10 percent. Consequently, at least for the case of tension the TM is sufficiently accurate. The degree of the anisotropy m is plotted in the Fig. 7. As expected in the latter stages of the process the response tends back to the isotropic.

A short excursion into dynamic problems [3,7] was made primarily to test the feasibility of the basic concepts. Despite radical simplifications the proposed model, simplified to bare bones, was still able to duplicate all major trends observed in experiments.

III.4 Uniaxial Compression

The mesomechanics of the damage evolution in uniaxial compression is a great deal more complicated than in the preceding case (see Brace and Bombolakis 1963, and Santiago and Hilsdorf 1973). In absence of tensile macro-stresses the interfacial cracks originally grow in the Mode II along the facet. After reaching the facet edge they kink in the direction of the compressive axis (see also M. Kachanov 1982, Horii and Nemat-Nasser 1985). These kinked cracks initially grow in a stable fashion with the increasing force until at a certain level of the axial compression they become unstable. The instability point strongly depends on the lateral confinement and, in fact, the failure mode itself changes beyond the transition point (Paterson 1978).

The studies performed within the scope of this project were restricted to the analyses of unconfined specimens [8,11]. The analysis was not as elegant as in the preceding case since the crack opening displacements of the kinked crack cannot be derived in analytical form even for a crack enveloped by an isotropic solid. Nevertheless, it was possible to determine approximate expressions which were in a close fit with the numerical computations.

The stress-strain curves for uniaxial compression [8] are plotted in Fig. 8 (no confinement) and Fig. 9 (small confinement). The apparent Poisson's ratio is depicted in Fig. 10 (no confinement) and Fig. 11 (with small confinement). In all cases the experimentally measured strains were replicated rather well by the proposed model. The absence of the data regarding the sieve grading of the tested specimens did not allow further parametric studies. This rather simple model was only partially checked using the SCM in [11].

IV. SUMMARY AND CONCLUSIONS

The principal objective of this program was to explore the least restrictive set of assumptions and simplifications leading to a constitutive theory for brittle deformation processes of concrete which: (i) will reflect the salient aspects of the physical phenomenon and (ii) still be simple enough for practical applications. These conflicting requirements are found to be satisfied by models which either completely or partially ignore the crack interaction but, in return, admit a closed-form, analytical solution.

In the case of homogeneous states of stress of unconfined brittle solids, the ultimate failure emphasizes runaway propagation of a single, critical defect at relatively low crack density. Consequently, the simplest model, totally ignoring the crack interaction should be expected to be of adequate accuracy.

In the presence of lateral confinement above the transition point at the incipient localization (just ahead of the apex of the stress-strain curve), the crack interaction cannot be any more neglected. Within this relatively short phase of the deformation process, the proposed models will underestimate the inelastic strain. The ensuing increase in accuracy in the determination of the inelastic strains alone will not justify the attendant increase in complexity. However, a rational estimate of the macro-failure must obviously incorporate direct interaction of closely-spaced defects.

At this initial stage of its development this mesomechanical analytical model offers ample encouragements for the future. The ability of the model in duplicating the experimental results without using additional 'fudge' constant presents a strong impetus for the conduction of new, more complex studies. In that sense the present achievements constitute a sound foundation and a basic building block for the establishment of a versatile predictive tool useful in the design of runways and protective structures in the future.

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1. 'On the Structure of the Phenomenological Damage Theories' in: Steel Structures: Recent Advances and Their Application to Design, ed. M. N. Pavlovic, Elsevier Publ. Co. London, pp. 417-438, 1986.

2. 'A Micromechanical Damage Model for Concrete' (with D. Fanella), Eng. Fracture Mech., 25, pp. 585-596, 1986. Reprinted in 'Mechanics of Damage and Fatigue' eds. S. R. Bodner and Z. Hashin, Pergamon Press, 1986.

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10. D. A. Fanella, 'A Micromechanical Continuous Damage Model for Plain Concrete,' CEMM Dept. Univ. of Illinois at Chicago, May 1986.

11. D. Sumarac, 'A Self-Consistent Model for Brittle Response of Solids', CEMM Dept., Univ. of Illinois at Chicago, June 1987.

Note: The references listed in Chapter VI are referred to in the text of the Report using brackets.

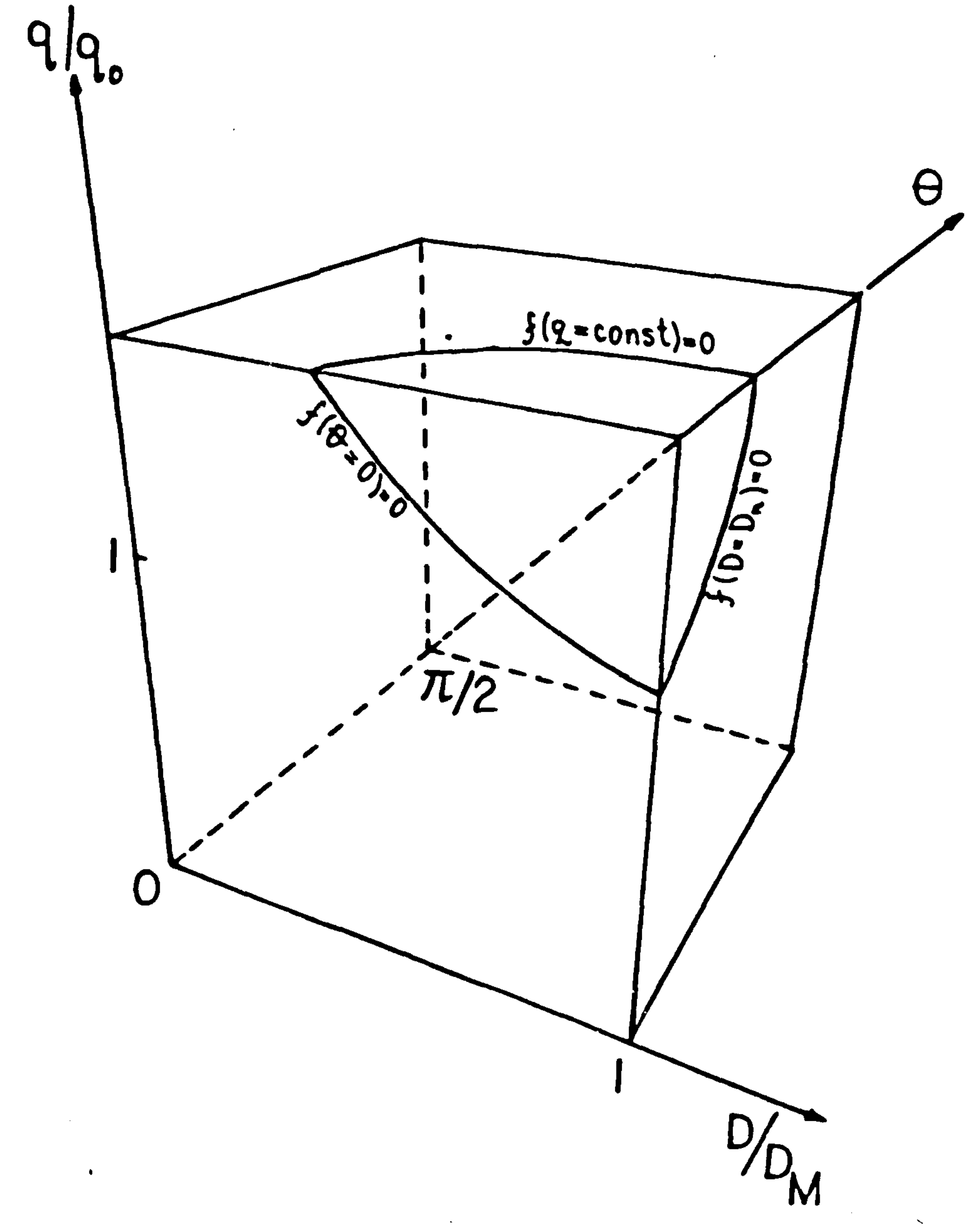

Fig. 1 The hypersurface $f = 0$ separating all cracks into those that were already destabilized (upper right corner) from those that retained their original size.

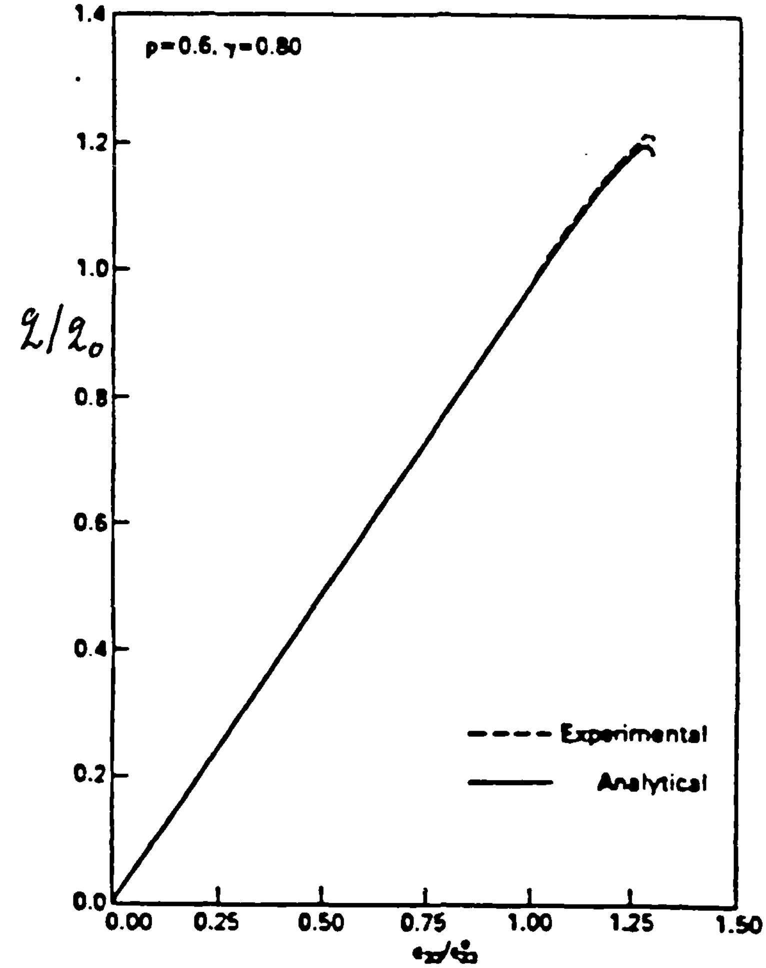

Fig. 2 Stress-strain curve for the case of uniaxial tension.

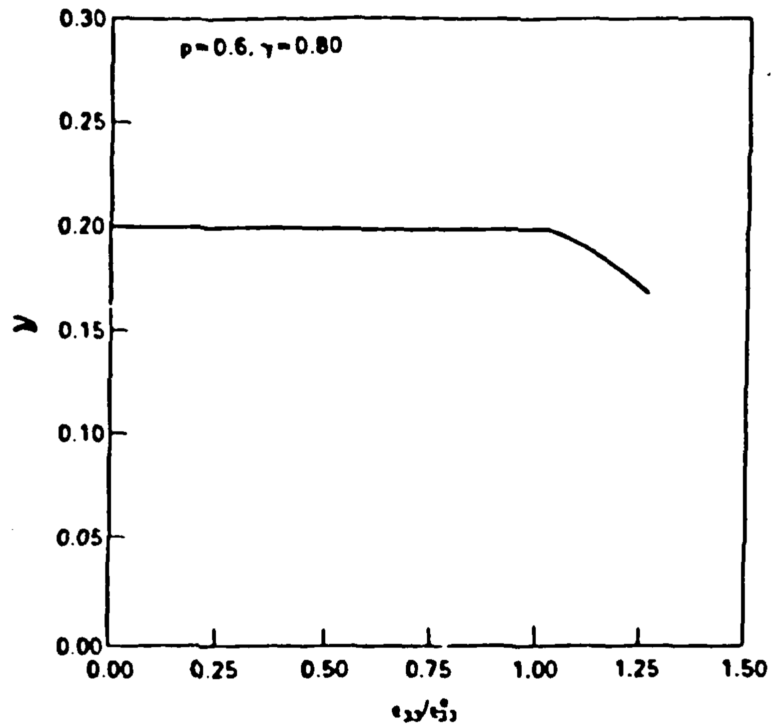

Fig. 3 The apparent Poisson's ratio for the case of uniaxial tension.

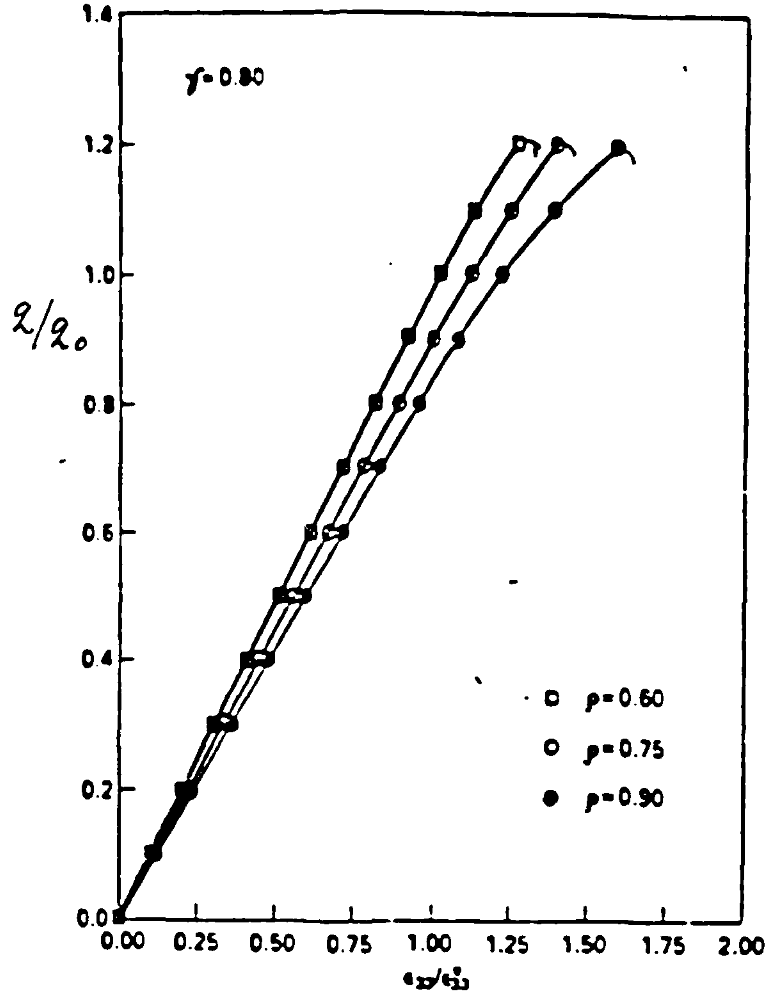

Fig. 4 The dependence of the total strain on the size of initial defects.

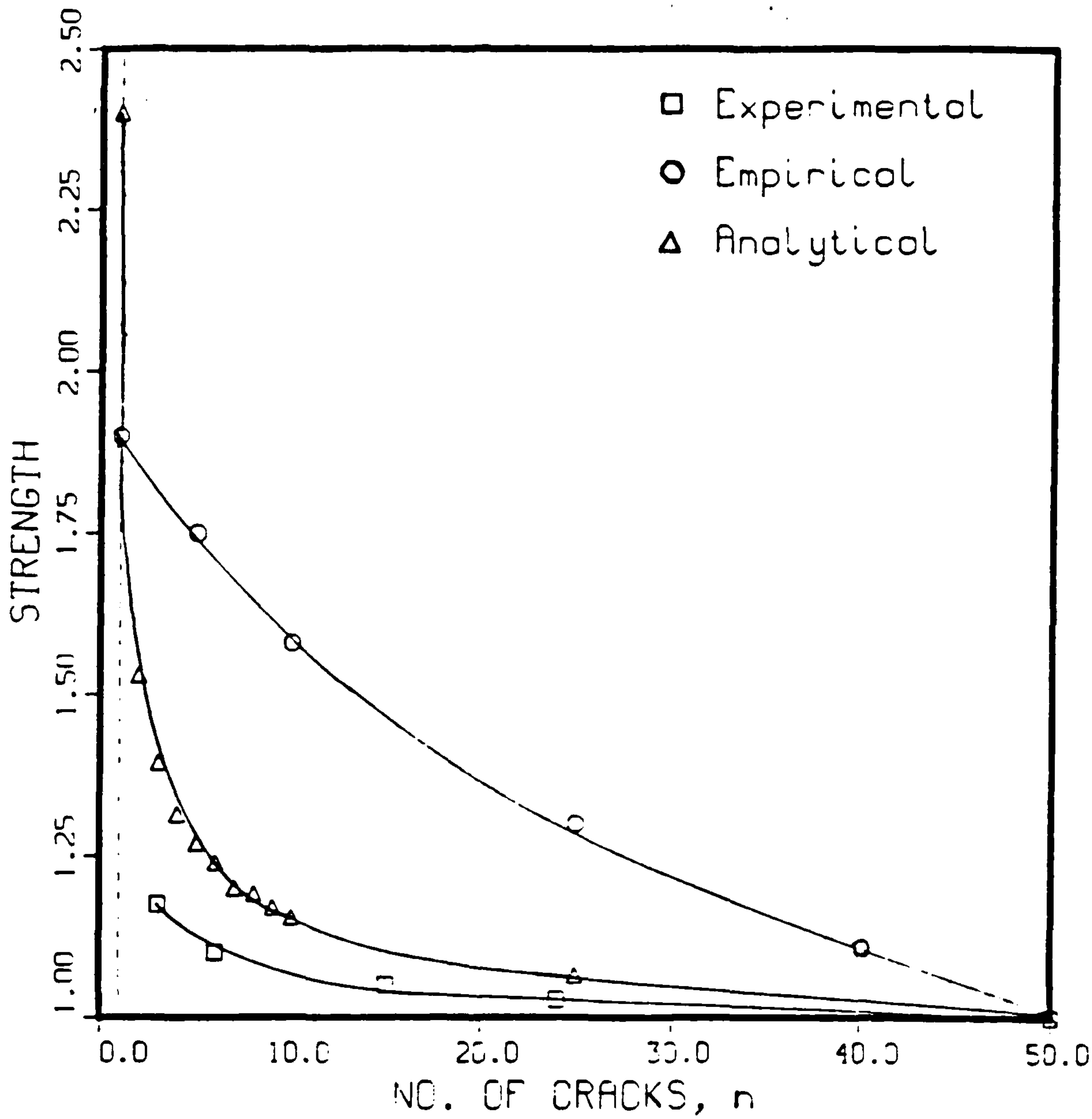

Fig. 5 **Size effect for specimens in uniaxial tension.**

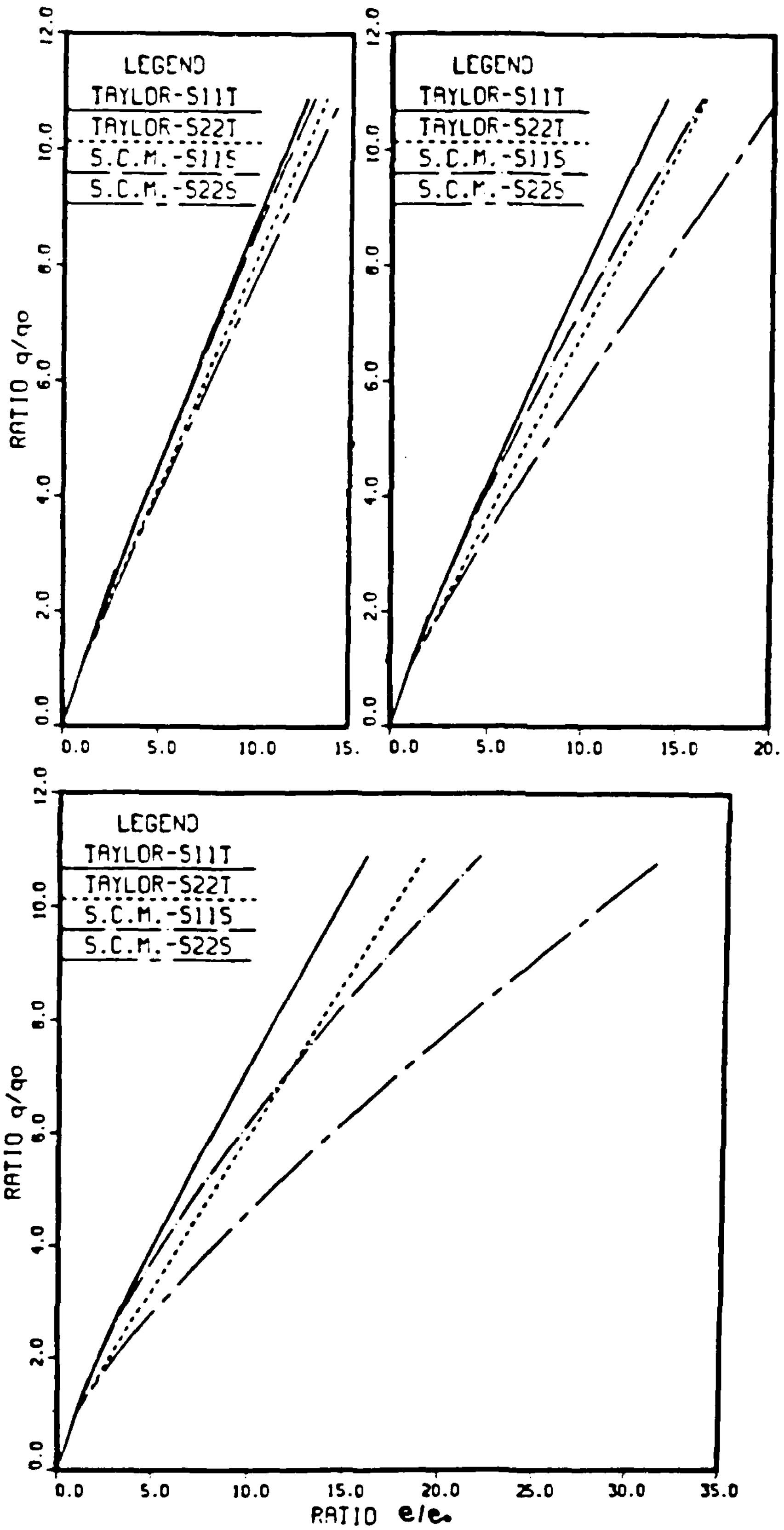

Fig. 6 The normalized stress-strain curves for uniaxial tension using SCM and TM for three levels of crack density.

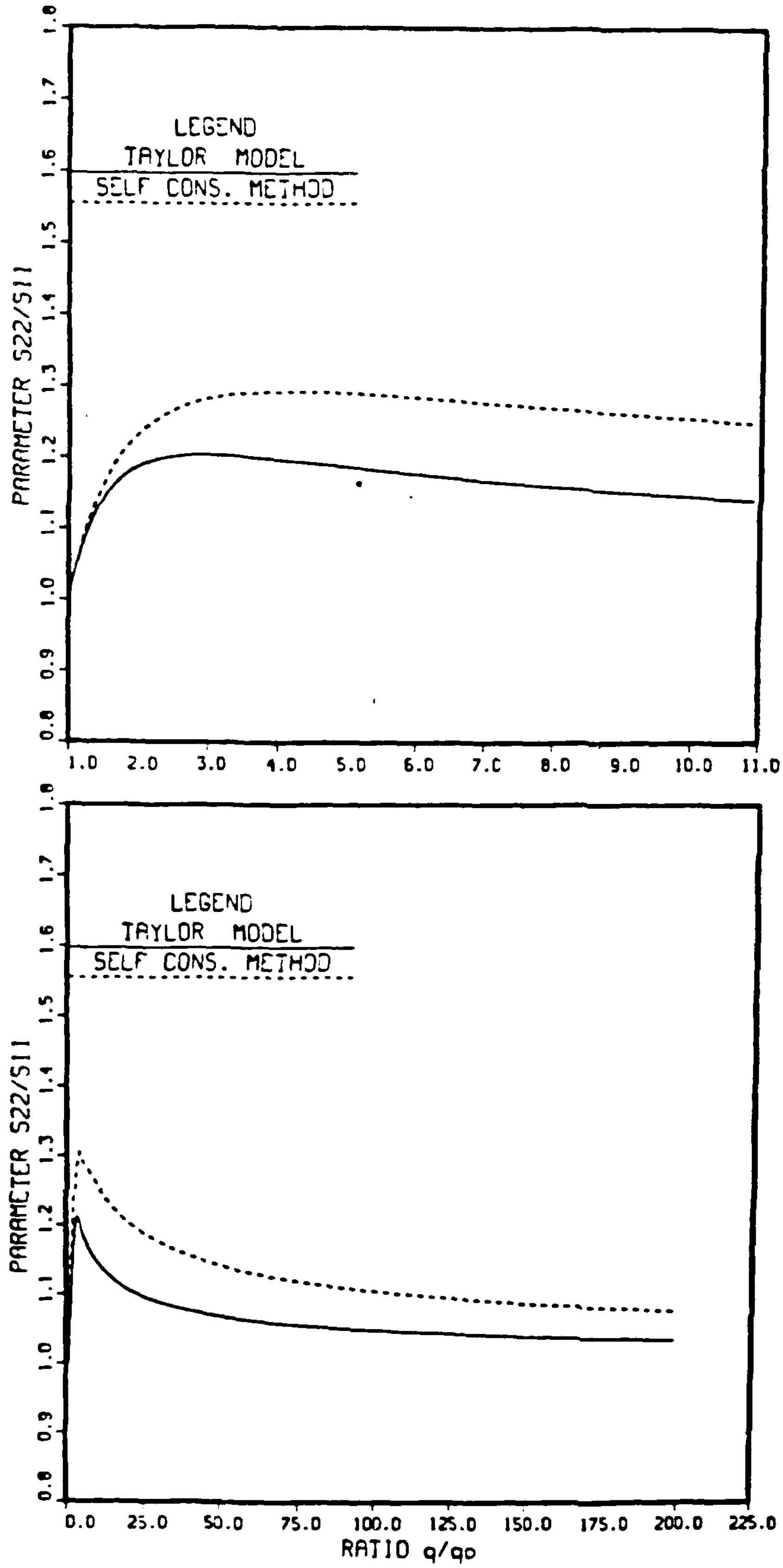

Fig. 7 The degree of anisotropy in the case of uniaxial tension.

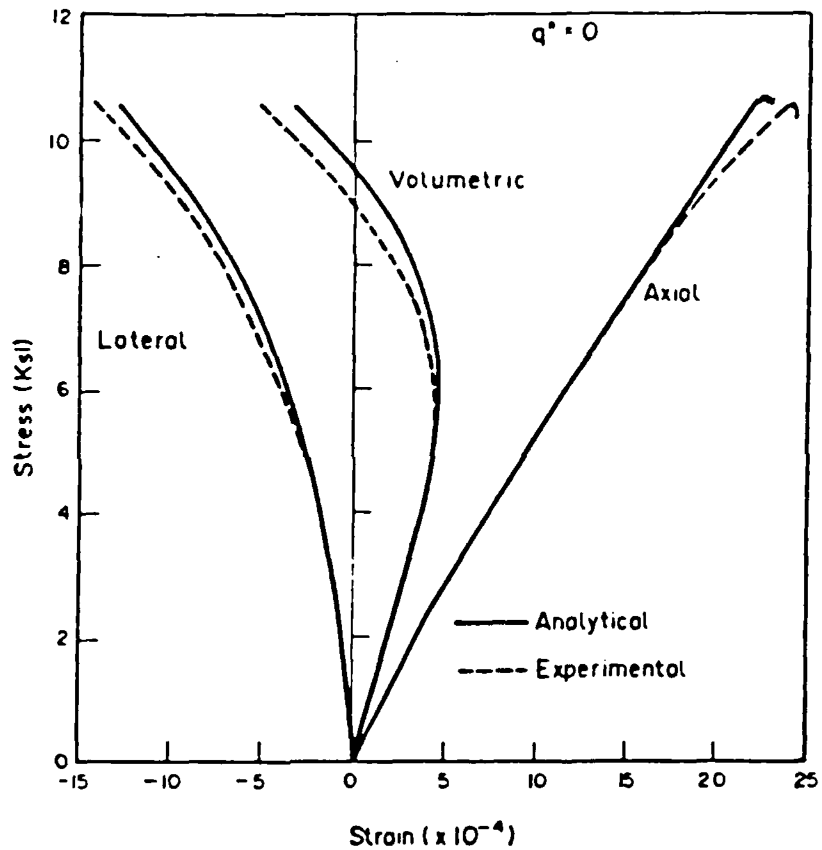

Fig. 8. Comparison of the analytical and experimental stress–strain curves for uniaxial compression (unconfined specimen).

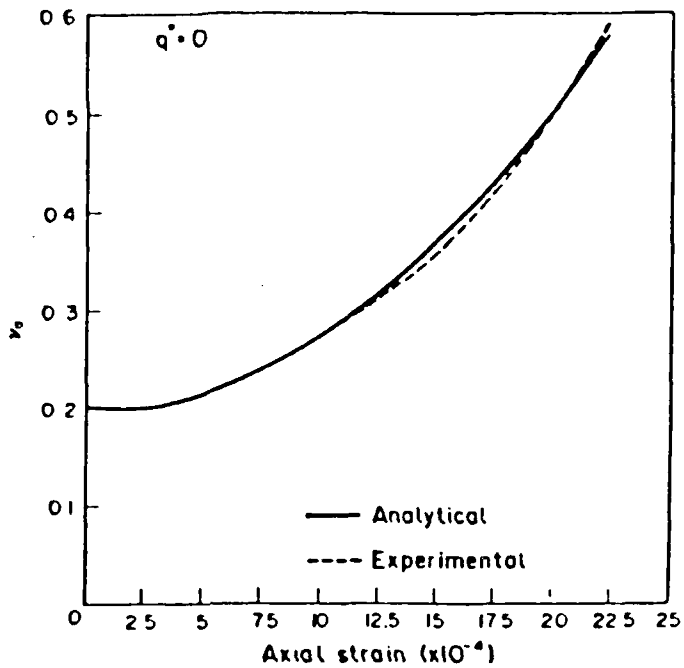

Fig. 9. Comparison of the analytical and experimental Poisson's ratio for uniaxial compression (unconfined specimen).

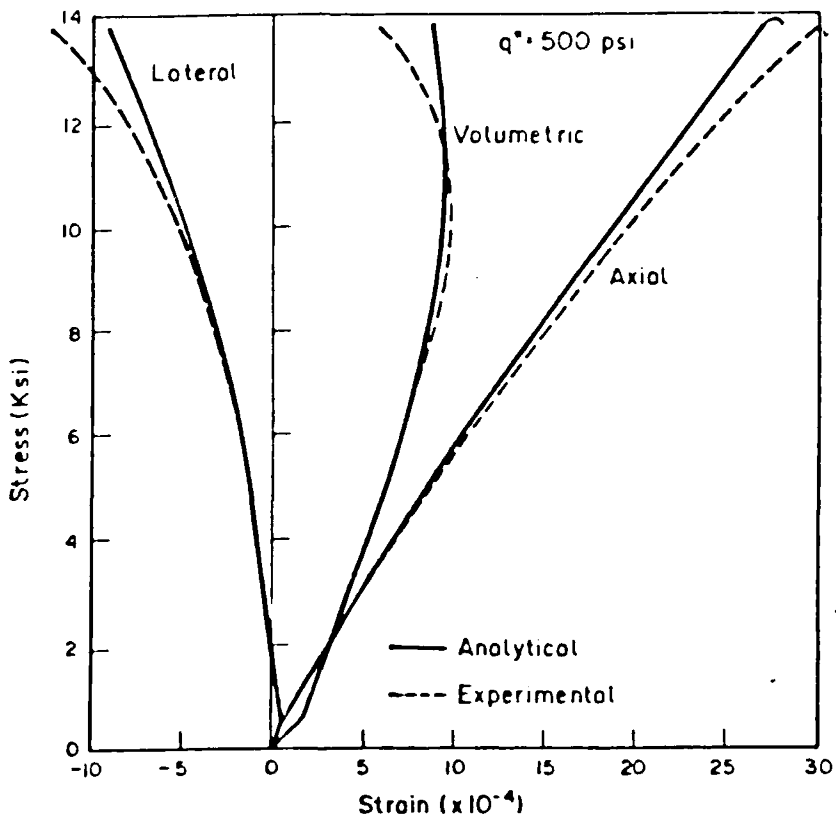

Fig. 10. Comparison of the analytical and experimental stress–strain curves for triaxial compression ($q^* = 500$ psi).

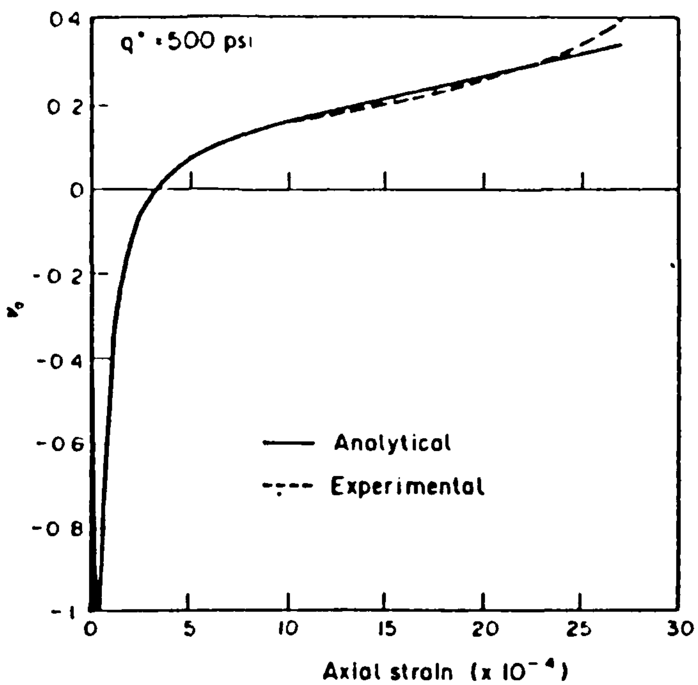

Fig. 11. Comparison of the analytical and experimental Poisson's ratio for triaxial compression ($q^* = 500$ psi).

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